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Checking calculations by estimation

Mathematical goals

This session is designed to help learners to:

- estimate the size of multiplication and division calculations, particularly when these involve numbers less than 1;
- check calculations by rounding and estimation.

It is advisable to complete Session 9 – *Exploring properties of number operations* and Session 10 – *The effect of multiplying and dividing* before attempting this session with your learners unless you are sure they have fully mastered the ideas contained in those sessions.

Materials required

For each pair or small group of learners you will need:

- Sheet 1 – *Checking work without a calculator*;
- Sheet 2 – *Checking work without a calculator: answer sheet*.

Notes

Suggested approach **Beginning the session**

Write the calculations in the first column below on an overhead projector slide, whiteboard or similar.

Ask learners to tell you how they can tell that each answer is incorrect.

Calculation	Examples of possible reasoning
$12 \times 13 = 135$	The answer should be greater than $12 \times 12 = 144$. The answer should end in a six because $2 \times 3 = 6$.
$29 \times 18 = 5222$	The answer should be roughly $30 \times 20 = 600$. The answer should be less than 600.
$162 \div 16 = 9$	The answer is about the right size because $160 \div 16 = 10$. But the answer should be bigger than 10, not smaller.
$4.2 \div 0.5 = 2.1$	The answer should be greater than 4.2. It means how many halves go into 4.2, so the answer should be just over 8.
$54 \times 0.7 = 378$	The answer should be less than 54, since 0.7 is less than 1. The answer should be about 30 or 40.
$5.6 \div 11.2 = 2$	The answer should be less than 1 because 11.2 is greater than 5.6. The question has been done the wrong way round.

Working in groups

Ask learners to work in pairs. Give each pair Sheet 1 – *Checking work without a calculator* and Sheet 2 – *Checking work without a calculator: answer sheet*. Read through the instructions on Sheet 1 with the learners carefully before they begin. Show them the example on Sheet 2.

Now ask learners to work in pairs to identify the wrong answers on Sheet 1 and give their reasons on Sheet 2. As you move from group to group, encourage them to:

- interpret each calculation in words;
- round the numbers appropriately;
- think about the effect of multiplying and dividing by numbers less than one.

If learners miss some incorrect answers, tell them that 15 of the 36 answers are incorrect. This should serve to reinvigorate the search.

Watch out for learners who use rules without understanding. For example, they reason that $15.4 \times 3.5 = 53.9$ is incorrect because 'the rule says that if there are two numbers to the right of the decimal point in the question, there must be two numbers to the right of the point in the answer'.

Ask each pair of learners to find one question they are sure is incorrect, and to prepare to explain their reasoning to the rest of the group.

Reviewing the learning

Conclude the session by asking pairs of learners to explain to the rest of the group how they know particular answers are incorrect. As they do so, point out the various strategies they are using.

Sample solutions

Incorrect	Typical reason
(1) $12 \times 18 = 144$	The answer should end in a 6 because 2×8 ends in a 6.
(2) $16 \div 32 = 2$	Division wrong way round. Should be 0.5.
(5) $160 \div 42 = 5$	$160 \div 40 = 4$. Answer should be less than 4.
(8) $99 \times 8 = 892$	$100 \times 8 = 800$. Answer should be less than 800.
(9) $39 \times 41 = 159$	$40 \times 40 = 1600$. Answer is too small.
(15) $8.3 \times 7.2 = 89.76$	$8 \times 7 = 56$. Answer should be only a little bigger than this.
(18) $6.4 \times 0.5 = 9.6$	Answer should be smaller than 6.4. (Half of 6.4) because you are multiplying by a number less than 1.
(19) $0.6 \times 23 = 138$	Answer should be smaller than 23. (Just over half 23) because you are multiplying by a number less than 1.
(22) $163 \times 3.1 = 50.3$	Answer should be greater than 163. (About 500)
(23) $0.2 \times 0.3 = 0.6$	Answer should be smaller than 0.2 and 0.3 because you are multiplying two number less than 1.
(26) $5.2 \div 10.4 = 2$	Division wrong way round. (Should be 0.5).
(28) $15 \div 100 = 0.015$	Answer should be 0.15.
(32) $8.2 \div 0.2 = 4.1$	Answer should be greater than 8.2 because you are dividing by a number less than 1. 'How many 0.2s in 8.2?' Answer 41.
(34) $0.8 \div 0.2 = 0.4$	Answer should be greater than 0.8 because you are dividing by a number less than 1. 'How many 0.2s in 0.8?' Answer 4.
(36) $0.4 \div 0.6 = 1.5$	Answer should be greater than 0.4 because you are dividing by a number less than 1. Division done wrong way round.

Sheet 1 – Checking work without a calculator



The questions and answers below are taken from some work completed by Sam. Imagine you are Sam's teacher.

Your job is to check the work **without doing all the questions again**.

You are not allowed to use a calculator.

Just use estimation and common sense to see what is wrong.

If you think an answer is correct, give it a tick.

If you think an answer is wrong, copy it onto Sheet 2 and explain how you know it is wrong.

Whole number multiplication and division		
(1) $12 \times 18 = 144$	(5) $160 \div 42 = 5$	(9) $39 \times 41 = 159$
(2) $16 \div 32 = 2$	(6) $315 \div 21 = 15$	(10) $190 \div 19 = 10$
(3) $50 \times 50 = 2500$	(7) $15 \times 14 = 210$	(11) $31 \times 68 = 2108$
(4) $212 \div 53 = 4$	(8) $99 \times 8 = 892$	(12) $42 \times 79 = 3318$

Decimal multiplication		
(13) $12 \times 1.5 = 18$	(17) $15.4 \times 3.5 = 53.9$	(21) $0.5 \times 0.4 = 0.2$
(14) $12.6 \times 5.7 = 71.82$	(18) $6.4 \times 0.5 = 9.6$	(22) $163 \times 3.1 = 50.3$
(15) $8.3 \times 7.2 = 89.76$	(19) $0.6 \times 23 = 138$	(23) $0.2 \times 0.3 = 0.6$
(16) $5.8 \times 100 = 580$	(20) $24 \times 0.4 = 9.6$	(24) $9.6 \times 10.5 = 100.8$

Decimal division		
(25) $12.6 \div 2.1 = 6$	(29) $99 \div 19.8 = 5$	(33) $24.36 \div 8.4 = 2.9$
(26) $5.2 \div 10.4 = 2$	(30) $6.4 \div 12.8 = 0.5$	(34) $0.8 \div 0.2 = 0.4$
(27) $24.8 \div 3.1 = 8$	(31) $2.2 \div 0.2 = 11$	(35) $100 \div 0.5 = 200$
(28) $15 \div 100 = 0.015$	(32) $8.2 \div 0.2 = 4.1$	(36) $0.4 \div 0.6 = 1.5$

